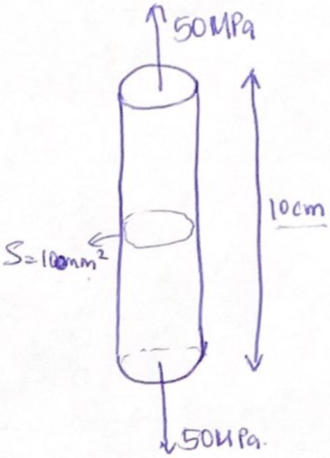
Exercise 04 – Task 1

Tension of a Cylinder – Hookes Law.



We find in experiment: $\Delta l = 2.5 \times 10^{-5}$ m

$\Delta S = -10^{-7} m^2$

Calculate: Youngs Modulus, E

Poissons Ratio, ν

Lame Parameters λ, μ

Bulk Modulus, K

$$\sigma = E \frac{\Delta l}{l}$$

Stress = Youngs Modulus · Strain

$$50\text{MPa} = E \frac{2.5 \times 10^{-5}}{0.10}$$

$$E = 200\text{GPa}$$

$$\nu = -\frac{\frac{\Delta S}{S}}{\frac{\Delta l}{l}} = \frac{\frac{-10^{-7}}{10^{-4}}}{\frac{2.5 \times 10^{-5}}{0.1}}$$

$$\nu = 0.3$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

$$\underline{\underline{\sigma}} = \overset{4}{\underline{\underline{C}}}\, \underline{\underline{\varepsilon}} \Rightarrow \sigma_{ij} = C_{ijkl}\, \varepsilon_{kl}$$

⇒ Voigt Notation

$E \rightarrow$ Elasticity Matrix (Stiffness Matrix).

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \\ \\ & E & \\ \\ \\ \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} \leftarrow \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ & \varepsilon_{22} & \varepsilon_{23} \\ & & \varepsilon_{33} \end{bmatrix}$$

Stress Tensor components

Small Strain Tensor components

For an isotropic linear elastic material, we have the components of the 4th order isotropic tensor given as

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

This leads to the following stiffness matrix.

$$[E] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

* Calculate $\boxed{E}$? The Stiffness Matrix.

On the other hand, the ~~com~~ relation between strain & stress can be given by

$$\varepsilon_{ij} = C^{-1}_{ijkl} \sigma_{ij}$$

This leads us to the compliance matrix, in the Voigt notation given by

↗ compliance matrix

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}$$

* Calculate the Compliance Matrix